Analytical study of coupling between subsystems of a vehicule NVH model

Eric Courteille, Lionel Leotoing, Frédéric Mortier, Eric Ragneau

The design of an automotive powertrain mounting system plays an important role in improving vehicle noise, vibration and harshness (NVH). One of the main problems encountered in the automotive design remains the isolation of the low frequencies vibrations of the engine from the rest of the vehicle. Several engine mounting schemes have been developed to deal with this problem. Most of these strategies stem from arranging the rigid body modes of the engine mounted on resilient supports to have certain coupled or decoupled characteristics. It is currently admitted in literature that a decoupled powertrain mounting system improves NVH characteristics. The signi£cant engine mass makes the right frequencies and modes arrangement a critical design decision. But it can not be stated that decoupling the on-ground rigid body modes of the engine will systematically reduce chassis vibrations. In this paper, a new analytical method is proposed to examine the mechanisms of coupling between the engine and the vehicle body structure. The analytical procedure enable to de£ne the domain of validity of the mounting schemes based on a 6 degrees-of-freedom engine model and to assess NVH improvement.

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1 Introduction

In vehicles, the engine mounts play an essential role for the NVH comfort. The main functions of these mounts (rubber or hydraulic) are to provide static supports for the engine and to isolate the vibrations of the engine from the rest of the vehicle. Thus, the modelling of these vehicle parts, becoming of a great complexity, constitutes an essential step for the NVH improvement. Besides, in addition to a good de£nition of the different vehicle parts, it is necessary to focus on the function of the subsystem in the whole vehicle by evaluating the main interactions with other components. Simulation of the engine mounting system at low frequency enable to de£ne optimal architecture, and to give the characteristics of the engine mounts necessary in terms of rigidity and damping. But, to carry out an engine mounting system layout theoretically, system data have to be provided. As a good de£nition of the rest of the vehicle is not available in early stages of the vehicle design process, some assumptions have to be made. Speci£cally, the model includes rigid body representations of the engine and the chassis with appropriate values for the location of the centers of gravity, masses, and moments of inertia. Therefore, only predictions in the lower frequency range are possible (< 50 Hz). Such a simulation model enable to assess the rigid body modes of the engine in the vehicle as well as to analyse the motion of the engine and the chassis under various engine operating conditions (idle, full load speed sweep) and road/wheel inputs.

While designing a powerplant mounting system, one of the main items to consider is the isolation of the engine vibration from the rest of the vehicle. Generally, this step is largely affected by other vehicle considerations such as packaging constraints and the need for common parts between vehicle platforms. Various analytical isolation schemes based on rigid body models simulation are employed to optimize the type of mounts, their number, location and own characteristics with respect to the overall characteristics of the engine mounting system. These strategies include natural frequency placement, torque axis mounting and elastic axis mounting [1]. The background theory of these techniques is widely described in literature [2, 3, 1]. All these studies consider the engine by its on-ground behavior, neglecting the effects of the chassis, exhaust subsystem, drive-shaft, wheel suspension . . .

Lately, researches have focused on the signi£cance of the rigid body modes alignment for on-ground engine to its in-vehicle behavior [4, 5]. These studies deal with the accuracy of NVH vehicle models and raise the problem of interactions between the different subsystems. Various powertrain models have been studied and their accuracy was discussed through a full vehicle model. By the evaluation of actual cases, the existence of these interactions have been clearly demonstrated. Nevertheless, no general formalism have been introduced to evaluate the limits of the modelling assumptions made during the development of the classical 6 d.o.f. engine mounting schemes.

The aim of the proposed method is to highlight and identify, through an analytical procedure, the relationships between the engine mounting schemes and the vehicle response characteristics. In the second section, the general equations of motion are reformulated using an original matrix, the coupling matrix introduced for coupled plates [6]. The characteristics of the coupling matrix, analyzed in the third section, enable to de£ne the domain of validity of the mounting schemes based on a 6 d.o.f. engine model and to assess NVH improvement. In the last section, the limitations of the current mounting strategies are discussed.

2 FORMULATION OF THE COUPLING PROBLEM

2.1 Modelling of the vehicle system

In derivation of the equations of motion to simulate dynamic behaviors of engine mount systems with supporting structures, a good modelling of a total vehicle system can consist in four subsystems : engine (powertrain), engine mounts, chassis and suspensions. Since small displacements can be assumed, engine is modelled as rigid body of time-invariant inertial matrix of dimension 6. The engine is supported by an arbitrary number of mounts to the vehicle chassis, also modelled as a rigid body elastically suspended (Figure 1).

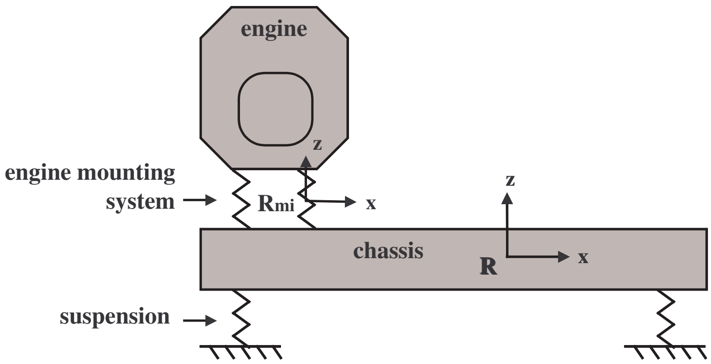

Figure 1: Engine mounting model

The mounts classically used in engine mounting application are rubber bonded to metal construction. It is possible to get better isolation effects than conventional rubber mount systems with a hydraulic engine mount. Elastomeric materials behave visco-elastically, thus engine mounts are represented by three mutually perpendicular sets of linear spring and corresponding viscous damper in parallel. No rotational stiffness of the mounts will be considered. The connection to the ground is also simply represented by four systems of linear spring and viscous damper in parallel at each wheel, characterized by their stiffness and damping coef£cients following the three directions of the vehicle frame coordinates $\mathcal{R}$ (Figure 1).

By assuming all of the elastic loadings from all mounts and suspensions, the total elastic loadings on the engine and chassis centers of gravity can be expressed through a generalized square stiffness matrix $\mathbf{K}$ of dimension 12 (1), resulting from the assembly of the stiffness elementary matrices of mounts and suspensions.

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}^e & -\mathbf{K}^{e\to c} \\ -\mathbf{K}^{c\to e} & \mathbf{K}^c \end{bmatrix} \tag{1}$$

The matrix $\mathbf{K}^{e\to c}$ is the engine's matrix of in¤uence on the chassis and reciprocally, $\mathbf{K}^{c\to e}$ is the chassis's matrix of in¤uence on the engine. Using a similar assembly procedure to the elastic loadings, the total damping loadings on the engine and chassis centers of gravity can be expressed by a generalized square stiffness damping matrix $\mathbf{C}$ of dimension 12.

2.2 Equations of motion

The previous assumptions have become standard practice [3] for the development of system simulation models of vehicles. Because of the broad band input that a vehicle encounters, simulations are generally performed in the frequency domain. The dynamic equations of motion of the engine and the car body structure can be written as the matrix form in the frequency domain :

$$\left(-\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K}\right)\mathbf{q}\left(\omega\right) = \mathbf{F}\left(\omega\right), \tag{2}$$

where $\mathbf{q}$ is the generalized vector of displacement, de£ned by combining translational $\mathbf{u}$ and rotational $\boldsymbol{\theta}$ displacements of the centers of gravity of the engine and the chassis (3). The superscripts e and c respectively stand for engine and chassis. The superscript j may referred to either the engine or the chassis.

$$\mathbf{q} = {}^t\left\{\begin{array}{cc} \mathbf{q}^e & \mathbf{q}^c \end{array}\right\} = {}^t\left\{\begin{array}{cccc} \mathbf{u}^e & \boldsymbol{\theta}^e & \mathbf{u}^c & \boldsymbol{\theta}^c \end{array}\right\} \tag{3}$$

The vector $\mathbf{F} = {}^t\left\{\begin{array}{cc} \mathbf{F}^e & \mathbf{F}^c \end{array}\right\}$ is the generalized external load vector. The external excitation is harmonic with known frequencies, amplitudes and phases. Engine excitation forces are applied to the engine at the center of the crankshaft location. The response to road inputs can be studied by applying forces or displacements at the suspension ground contact locations in the vehicle model.

The matrix $\mathbf{M}$ is the generalized mass matrix of the system (4) :

$$\mathbf{M} = \left[\begin{array}{cc} \mathbf{M}^e & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^c \end{array}\right] = \left[\begin{array}{cccc} \mathbf{M}^e_f & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^e_\tau & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}^c_f & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}^c_\tau \end{array}\right], \tag{4}$$

$$\text{with } \mathbf{M}^j_f = diag(m^j),$$

where m^j is the mass of the rigid body j and $\mathbf{M}^j_\tau$ its inertia matrix.

If a structural damping matrix $\mathbf{H}$ is considered, viscous damping term $j\omega\mathbf{C}$ may be replaced by the structural damping term $j\mathbf{H}$. In the following sections, a complex stiffness ($\overline{\mathbf{K}}$) is then used to model the dynamic behavior of the isolators (5).

$$\left(-\omega^2\mathbf{M} + \overline{\mathbf{K}}\right)\mathbf{q}\left(\omega\right) = \mathbf{F}\left(\omega\right) \tag{5}$$

2.3 Introduction of the coupling matrix

The response of the engine and chassis centers of gravity can be calculated through the solving of equation (2). Then the complex matrix inversion of equation (6) is classically used.

$$\left\{\begin{array}{c} \mathbf{q}^e \\ \mathbf{q}^c \end{array}\right\} = \left[\begin{array}{cc} \left(\overline{\mathbf{K}}^e - \omega^2\mathbf{M}^e\right) & -\overline{\mathbf{K}}^{e\to c} \\ -\overline{\mathbf{K}}^{c\to e} & \left(\overline{\mathbf{K}}^c - \omega^2\mathbf{M}^c\right) \end{array}\right]^{-1} \left\{\begin{array}{c} \mathbf{F}^e \\ \mathbf{F}^c \end{array}\right\} \tag{6}$$

The inversion of the matrix of impedance is numerically commonplace. Nevertheless, this method of resolution prevent from understanding the coupling phenomena between the engine and the chassis. From the traditional equation of motion (6), one can isolate a matrix presenting only terms related to the coupling from the two bodies (7).

$$\begin{bmatrix} \mathbf{I} & -\left(\overline{\mathbf{K}}^e - \omega^2\mathbf{M}^e\right)^{-1}\overline{\mathbf{K}}^{e\to c} \\ -\left(\overline{\mathbf{K}}^c - \omega^2\mathbf{M}^c\right)^{-1}\overline{\mathbf{K}}^{c\to e} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{q}^e \\ \mathbf{q}^c \end{Bmatrix} = \begin{Bmatrix} \left(\overline{\mathbf{K}}^e - \omega^2\mathbf{M}^e\right)^{-1}\mathbf{F}^e \\ \left(\overline{\mathbf{K}}^c - \omega^2\mathbf{M}^c\right)^{-1}\mathbf{F}^c \end{Bmatrix} \quad (7)$$

For the sake of physical meaning of the coupling mechanism, the term $\left(\overline{\mathbf{K}}^e - \omega^2\mathbf{M}^e\right)^{-1}\mathbf{F}^e$ in equation (7) represents the displacement of the engine subjected to his own excitation when the chassis is blocked (suspensions with in£nite stiffnesses). This con£guration represents the on-ground behavior of the engine (Figure 2-(a)).

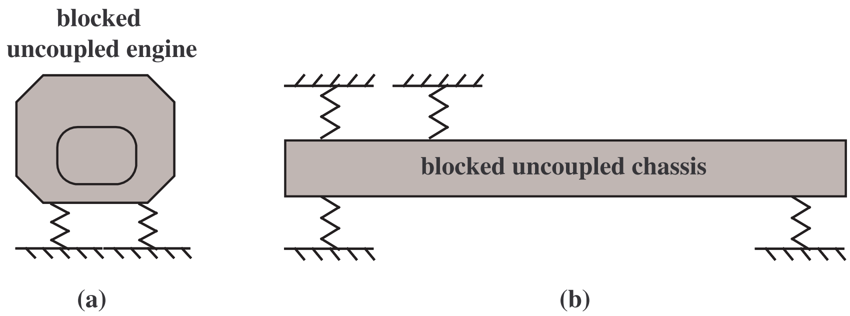

Figure 2: Blocked uncoupled bodies

The term $\left(\overline{\mathbf{K}}^c - \omega^2\mathbf{M}^c\right)^{-1}\mathbf{F}^c$ represents the displacement of the chassis subjected to his own excitation when the engine is blocked (null displacements) (Figure 2-(b)). This con£guration however do not represent a realistic behavior. One expresses the two con£gurations by the generalized vector displacement of the blocked uncoupled bodies ${}^t\left\{ \mathbf{q}_0^e \quad \mathbf{q}_0^c \right\}$ (8).

$$\begin{Bmatrix} \mathbf{q}_0^e \\ \mathbf{q}_0^c \end{Bmatrix} = \begin{Bmatrix} \left(\overline{\mathbf{K}}^e - \omega^2\mathbf{M}^e\right)^{-1}\mathbf{F}^e \\ \left(\overline{\mathbf{K}}^c - \omega^2\mathbf{M}^c\right)^{-1}\mathbf{F}^c \end{Bmatrix} \quad (8)$$

While revealing the vector displacement of the coupled systems, the equation (7) takes a form such that a coupling matrix $\mathbf{D}$ appears [6] (9).

$$\begin{Bmatrix} \mathbf{q}^e \\ \mathbf{q}^c \end{Bmatrix} = \begin{Bmatrix} \mathbf{q}_0^e \\ \mathbf{q}_0^c \end{Bmatrix} + \mathbf{D}\begin{Bmatrix} \mathbf{q}^e \\ \mathbf{q}^c \end{Bmatrix} \quad (9)$$

$$\text{with } \mathbf{D} = \begin{bmatrix} \mathbf{0} & \left(\overline{\mathbf{K}}^e - \omega^2\mathbf{M}^e\right)^{-1}\overline{\mathbf{K}}^{e\to c} \\ \left(\overline{\mathbf{K}}^c - \omega^2\mathbf{M}^c\right)^{-1}\overline{\mathbf{K}}^{c\to e} & \mathbf{0} \end{bmatrix}$$

Each one of the coupling matrix terms represents the action of the engine mass displacement (respectively chassis) on the chassis mass displacement (respectively engine). The matrix of coupling describes the exchanges between the masses independently of the external excitation. The coupling matrix, studied in more details in the following part, is then a practical solution to predict the global behavior of a system starting from the behavior of the isolated subsystems.

3 COUPLING MATRIX

This section is dedicated to the analysis of the coupling matrix by extracting its eigenvalues. The value of these intrinsic characteristics permits to identify different zone in which the degree of coupling between the engine and the chassis can be evaluated through an original parameter : the coupling order.

3.1 Eigenvalues and eigenvectors of coupling

The eigenvalues of coupling $\lambda_r(\omega)$ ($r = 1, .., 12$) and the eigenvectors of coupling $\boldsymbol{\varphi}_r(\omega)$ can be extracted from the coupling matrix (10).

$$\det\left(\mathbf{D} - \lambda_r(\omega)\mathbf{I}\right) = 0 \tag{10}$$

The Figure 3 presents the evolution of the spectral radius of the coupling matrix $\mathbf{D}$ versus frequency for three representative models of front wheel drive cars (a 3, 4 and 6 cylinder engine). The spectral radius of $\mathbf{D}$ is de£ned by $\rho\left(\mathbf{D}\right) = \max_r |\lambda_r|$.

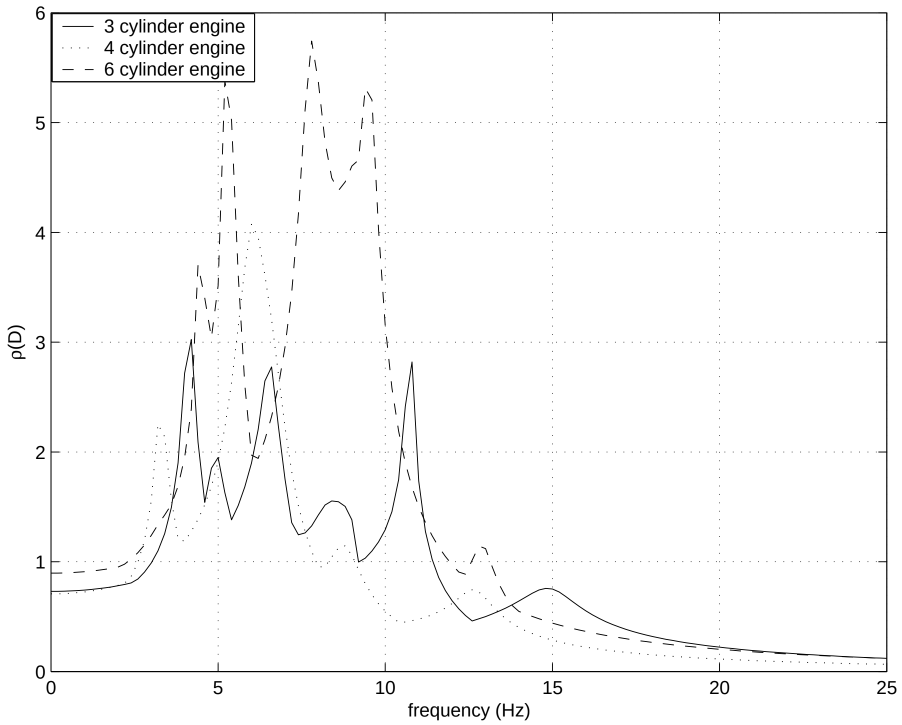

Figure 3: Spectral radius of the coupling matrix $\mathbf{D}$ of 3 models of front wheel drive cars (a 3, 4 and 6 cylinder engine)

One can see in this £gure that resonant peaks appear for the spectral radius and at the resonance frequencies of the uncoupled blocked bodies (Table 1).

This property of the spectral radius of $\mathbf{D}$ is clearly demonstrated by rewriting the determinant of the coupling matrix (11).

$$\det\mathbf{D} = \frac{\det\overline{\mathbf{K}}^{e\rightarrow c}\det\overline{\mathbf{K}}^{c\rightarrow e}}{\det\left(\overline{\mathbf{K}}^{e} - \omega^2\mathbf{M}^e\right)\det\left(\overline{\mathbf{K}}^{c} - \omega^2\mathbf{M}^c\right)} = \prod_r \lambda_r \tag{11}$$

Engine frequencies (Hz)	1st	2nd	3rd	4th	5th	6th
3 cylinder engine	4.12	4.92	6.55	9.14	10.73	14.89
4 cylinder engine	3.28	5.99	6.57	8.80	12.58	12.67
6 cylinder engine	4.47	5.25	7.69	9.06	9.52	12.90

Chassis frequencies (Hz)	1st	2nd	3rd	4th	5th	6th
3 cylinder engine	3.48	4.89	5.01	5.32	5.62	7.07
4 cylinder engine	3.72	4.72	5.05	5.58	5.76	6.64
6 cylinder engine	3.43	3.98	4.54	4.86	5.45	7.76

Table 1: Frequencies of the rigid body modes of the blocked uncoupled engine and chassis

At the resonance frequencies of the uncoupled blocked bodies, $(\det(\overline{\mathbf{K}}^e - \omega^2\mathbf{M}^e))$ or $(\det(\overline{\mathbf{K}}^c - \omega^2\mathbf{M}^c))$ is null, then the coupling eigenvalues product and respectively the spectral radius become in£nite. The increase in the damping of the connections smoothes these resonant peaks. Moreover, the frequencies of the peaks are located in the frequency band de£ned by the eigenvalues of the coupled bodies [7] (Table 2).

Frequencies (Hz)	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
3 cylinder engine	3.01	3.78	4.39	4.84	5.98	6.11	6.34	6.55	8.09	9.54	11.41	15.34
4 cylinder engine	2.58	3.39	3.78	4.01	4.88	5.02	5.43	7.15	7.97	9.28	12.67	12.95
6 cylinder engine	2.28	2.67	3.02	3.39	4.07	4.14	5.31	6.68	7.85	9.35	11.93	13.28

Table 2: Frequencies of the rigid body modes of the engine on mounting system

To support the engine weight and to avoid interference between the engine and the chassis during limit running conditions such as bumps and sudden brakes, a minimum level of stiffness is necessary for the engine mounts. The frequency range of the rigid body modes of suspension are then located between 3 and 20 Hz according to the mass of the engine.

3.2 Hypothesis on the equations of motion

The coupling matrix describes the contribution of one system on the other. One rewrites the equation (9) connecting the displacement of the systems coupled with the displacement of the blocked uncoupled bodies (12).

$$\left\{ \begin{array}{c} \mathbf{q}^e \\ \mathbf{q}^c \end{array} \right\} = (\mathbf{I} - \mathbf{D})^{-1} \left\{ \begin{array}{c} \mathbf{q}_0^e \\ \mathbf{q}_0^c \end{array} \right\} \tag{12}$$

The components resulting from the excitation $(\mathbf{q}_0^j)$ are isolated from the terms of the coupling matrix, intrinsic with the phenomena of coupling. If $\|\mathbf{D}\| < 1$, where $\|.\|$ is the Frobenius norm or one of the p-norms, then $(\mathbf{I} - \mathbf{D})$ is nonsingular and :

$$(\mathbf{I} - \mathbf{D})^{-1} = \sum_{n=0}^{\infty} \mathbf{D}^n. \tag{13}$$

Thus, the equation (12) can be written as follow while $\|\mathbf{D}\| < 1$:

$$\left\{ \begin{array}{c} \mathbf{q}^e \\ \mathbf{q}^c \end{array} \right\} = \sum_{n=0}^{\infty} \mathbf{D}^n \left\{ \begin{array}{c} \mathbf{q}_0^e \\ \mathbf{q}_0^c \end{array} \right\}. \tag{14}$$

The spectral radius of the 12-by-12 matrix of coupling $\mathbf{D}$, de£ned in the previous section, gives the lower bound of the all norms matrix of $\mathbf{D}$ (15).

$$\rho(\mathbf{D}) \leq \|\mathbf{D}\| \tag{15}$$

As shown before, the resonant peaks of the spectral radius appear at the resonance frequencies of the uncoupled blocked bodies (Figure 3), *i.e.* between 3 and 16 Hz (Table 2). The £gure 4 shows that beyond 16 Hz, the value of the spectral radius decrease gradually for tending toward zero. From (15), the development (14) can only be valid apart from resonant peaks of the coupling eigenvalues, *i.e.* apart from the domain of appearance of the eigenvalues of the blocked uncoupled bodies.

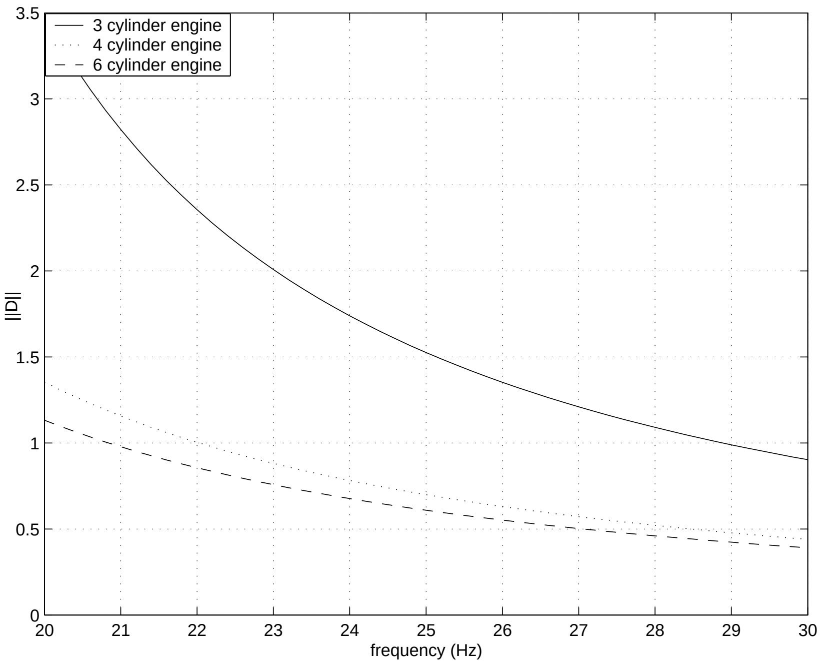

Figure 4: Frobenius norm of the coupling matrix $\mathbf{D}$ of 3 models of front wheel drive cars (a 3, 4 and 6 cylinder engine)

As presented in £gure 4, the assumption is valid from approximately 22 Hz for the 4 and 6 cylinder engine models. For the 3 cylinder engine model, the assumption is valid from approximately 30 Hz because the resonance frequencies of the uncoupled blocked bodies are slightly higher.

3.3 Coupling order

In an adequate interval of frequencies, *i.e.* when the value of the norm of the coupling matrix is negligible in front of the unit, the development of equation (14) can be restricted to a weaker order (16).

$$\left\{ \begin{array}{c} \mathbf{q}^e \\ \mathbf{q}^c \end{array} \right\} \approx \sum_{n=0}^{N} \mathbf{D}^n \left\{ \begin{array}{c} \mathbf{q}_0^e \\ \mathbf{q}_0^c \end{array} \right\} \tag{16}$$

The parameter N is called "coupling order", it expresses the coupling level between the two rigid bodies for the domain of frequency studied. The evaluation of N is essential for the global analysis in order to obtain simplistic relations between coupled system displacements and blocked uncoupled bodies ones. The last part of this paper notably illustrates the interests of low coupling orders.

The traditional mounting system design strategies require the rigid body modes of the on-ground engine. Theses strategies involve only engine rigid body mode arrangements for ”shaping” the engine vibratory behavior, *i.e.* the vector $\mathbf{q}_0^e$. According to the domain of frequency studied, the phenomena of interactions between rigid bodies can drastically modify the vibratory responses of the vehicle. The internal vibroacoustic comfort in automobiles is directly controlled by the chassis acceleration. In a preliminary design phase, a 6 d.o.f. model is indispensable to conduct an engine rigid body mode analysis. But a higher level NVH model that contains engine and chassis on suspension should be used to understand the interactions between the rigid bodies thanks to the general expression (9) or to the expression (16) valid in the isolation band, where $\|\mathbf{D}\| < 1$. These expressions enable to examine the domain of validity of the modelling assumptions and pro£t towards classical engine mounting strategies on the NVH improvement.

4 Numerical evaluation of the coupling order at idle

4.1 3-cylinder engine

The inputs for a 3-cylinder engine are particularly low in frequency at idle ; the combustion forces are 1.5 order (17.5 Hz at 700 rpm) and the unbalance moments are £rst order (11.6 Hz at 700 rpm). Therefore, the fundamental frequency of the excitation is in the frequency range of the suspension modes, where the coupling eigenvalues present resonant peaks. In this range, the interaction between the rigid bodies is high (Figures 3 and 4) and the coupling with the chassis will strongly modify the vibratory response of the engine.

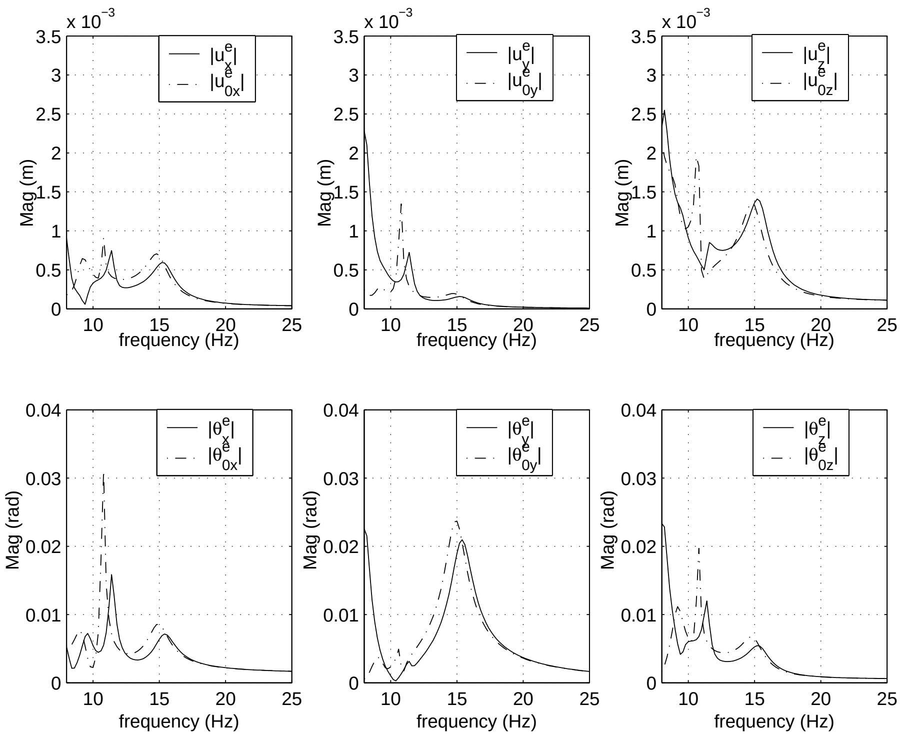

Figure 5: Frequency response of the engine center of gravity of the 3 cylinder engine model in the global coordinate system $\mathcal{R}$ for idle excitation

For this engine type, the development (16) is not valid and no coupling order can be de£ned at idle.

4.2 4-cylinder engine

This is in contrast with 4 and 6 cylinder engines for which an evaluation of the coupling order can lead to an analytical study of the coupling phenomena. The fundamental frequency of the excitation is in the zone of £ltering, beyond the frequencies of the rigid body modes. Indeed, at the second order, *i.e.* 25 Hz for a 4 cylinder engine at 750 rpm, and at the third order, *i.e.* 40 Hz for a 6 cylinder engine at 800 rpm, the value of the norm of the coupling matrix is small in front of the unit (Figure 4).

The £gure 6 represents the decoupled frequency response of the engine center of gravity of the 4 cylinder engine model for different values of the coupling order N. The engine response at the £rst order development, which corresponds to the blocked uncoupled engine response $\mathbf{q}_0^e$ because of the form extra-diagonal of $\mathbf{D}$, is quite different from the in-vehicle engine response ($N \longrightarrow \infty$).

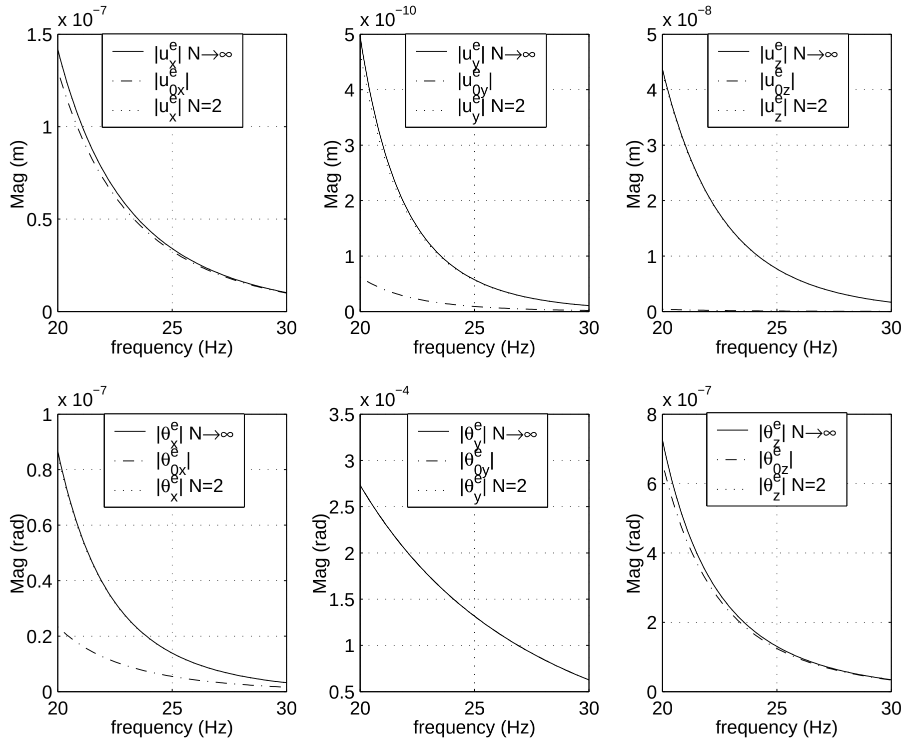

Figure 6: Decoupled frequency response of the engine center of gravity of the 4 cylinder engine model in the global coordinate system $\mathcal{R}$ with only crankshaft torque excitation

Then the development can be restricted to the second order for the engine displacement (Figure 6) since the difference between the two plots ($N = 2$) and ($N \longrightarrow \infty$) never exceeds 1%. In the same way, the £gure 7 represents the decoupled frequency response of the chassis center of gravity for the 4 cylinder engine model for different values of N. Then, for the frequency domain studied, a £rst coupling order leads to a good approximation of the chassis response. For the global system, a second coupling order is therefore suf£cient :

$$\left\{ \begin{array}{c} \mathbf{q}^e \\ \mathbf{q}^c \end{array} \right\} \approx \left(\mathbf{I} + \mathbf{D} + \mathbf{D}^2\right) \left\{ \begin{array}{c} \mathbf{q}_0^e \\ \mathbf{q}_0^c \end{array} \right\}. \tag{17}$$

According to the equation (17), with $\mathbf{q}_0^c = \mathbf{0}$, the chassis displacements are given by :

$$\mathbf{q}^c \approx \left(\overline{\mathbf{K}}^c - \omega^2 \mathbf{M}^c\right)^{-1} \overline{\mathbf{K}}^{c \rightarrow e} \mathbf{q}_0^e. \tag{18}$$

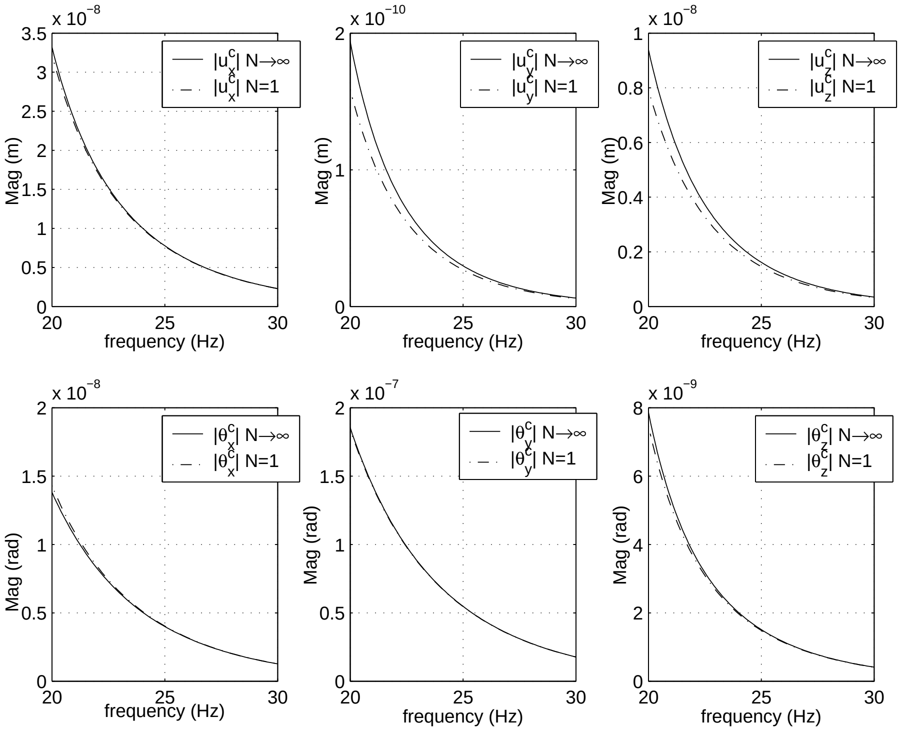

Figure 7: Decoupled frequency response the chassis center of gravity of the 4 cylinder engine model in the global coordinate system $\mathcal{R}$ with only crankshaft torque excitation

4.3 Automotive powerplant isolation strategies

The strategies currently used in industry analyze, under the modal approach, the harmonic response of the engine on resilient supports attached to ground [1, 8]. The modal analysis of this six degrees of freedom (d.o.f.) model is interesting insofar as the response to an excitation is calculated and interpreted according to the position in frequency and to the form of the modes. The usual strategy moves the rigid body natural frequencies of the engine away from the frequencies of the input sources to avoid resonances [9, 10].

By manipulating the rigid body modes of the on-ground engine, the torque roll axis decoupling and the elastic axis decoupling methods attempt to shape the response with the aim of minimizing the vibrations. In these common methods, it is hypothesized that the disturbances transferred to the car body structure can be reduced by conditioning the engine mounting system such that the rigid body mode of the engine are decoupled. The torque roll axis (TRA), is de£ned as the resulting £xed axis of rotation of an unconstrained three dimensional rigid body when a torque is applied along any axis. Geck and Patton [2] give a mathematical proof for the conditions which ensure that the engine pulsating torque excites only one engine mode. Singh and Jeong [3] demonstrate from the axioms of Geck and Patton that when the constant direction of the response becomes a rigid body mode of the on-ground engine, then the response is a rotation around the TRA, so with a constant direction. The torque roll axis decoupling strategy controls the displacement of the uncoupled blocked engine. The on-ground engine has a frequency response only in the TRA direction with the crankshaft torque variation excitation in all frequency range. The design objective is to reduce vehicle vibration in certain frequency range with respect to idle engine excitations. At idle, only the engine is directly excited, so the uncoupled blocked chassis has no displacements ($\mathbf{F}^c = 0$ then $\mathbf{q}_0^c = \mathbf{0}$). However the vibratory behavior of the vehicle cannot be limited to this six d.o.f model and the coupling effects of the chassis can be very important.

Then the use of the torque roll axis decoupling strategy for a 3-cylinder engine, already applied by Saitoh

[11], is debatable because the engine behavior in the vehicle can be very different from that on the ground (Figure 5, up to 15 Hz). A numerical analysis must be done thanks to equation (9) to obtain the hypothetic bene£ts on the chassis response by a puri£cation of the engine vibratory response.

Conversely, the issue of the torque roll axis strategy on the mounting system of 4 cylinder front wheel drive cars can be analytically analyzed by equation 18. A £rst coupling order leads to a simple relation between the on-ground engine response and the vehicle behavior. Such an expression enable to adjust easily the engine mounting system characteristics for a vehicle improvement.

5 Conclusions

This study was initiated with a desire to determine the signi£cance of the powertrain rigid-body modes for on-ground system to its in-vehicle NVH behaviour.

The current engine mounting strategies examined the rigid-body modes of the powertrain as it would sit on the mounts attached to the ground, *i.e.* neglecting the effect of the chassis. To predict correctly the issue of the traditional engine mounting strategies in terms of improvement of the dynamic chassis responses, it is essential to be able to analyze the phenomena induced by the coupling, and this for the whole of the excitation frequencies. The complexity of the harmonic response of the powertrain mounted on engine mounts in a vehicle cannot be understand using the traditional equations of motion.

The general equations of motion are reformulated by using the coupling matrix, intrinsic with the suspended bodies and independent of the external excitation. The coupling matrix constitutes the starting point of the analysis of the traditional engine mounting strategies and the order of coupling enable to de£ne their £eld of validity in frequency.

The use of a simple rigid-body representation of the engine and the chassis enable to concentrate only on the physique of the coupling problem. It is obvious that such a simplicity occults the effects of chassis ¤exibility or the modal properties of the cradle on the dynamic response.

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